

Modern status of magnetic monopoles

Yu.A. Kurochkin¹, Yu. Kulchitsky², I.S. Satsunkevich¹, Dz.V. Shoukovy^{1*},
V.V. Makhnatch¹, N.A. Rusakovich²

¹ Institute of Physics, National Academy of Sciences of Belarus

² Joint Institute for Nuclear Research, Dubna, Russia

Abstract

The modern theoretical status of magnetic monopole and experimental limits on the monopole mass are considered. We use two-photon production of monopole-antimonopole pair as instrument in estimation of quality for the search for magnetic monopole at accelerators. In the assumption that the monopole spin is equal 0,1/2,1, the monopole-antimonopole pair production cross section by this mechanism at LHC energies is estimated and analyzed.

As an example, for monopole spin is equal 1/2 the comparison of the $\gamma\gamma$ production total cross section for monopole-antimonopole pair and Drell-Yan total cross section at Tevatron energies is presented. In the case $\gamma\gamma$ pair production mechanism a mass limit $M > 455$ GeV for elastic $\bar{p}p$ scattering is set. We argue that all mass limits have not any sense based on quasitheoretical consideration up to now. Only experimental bounds are to be used as real indicator of our modern knowledge on monopoles.

1 Introduction

One of the outstanding questions in modern physics is the magnetic monopole problem. This problem has a very long history. The military engineer Pierre de Maricourt [1] in 1269 year was breaking magnets tried to separate their poles. In 1894 year P. Curie [2] assumed the existence of single magnetic poles. At last, after birth of quantum mechanics in 1931 Dirac [3] proposed that particles carrying magnetic charge, or magnetic monopoles, should exist. Dirac showed that the phase unobservability in quantum mechanics permitted singularities manifested as sources of magnetic fields, just as point electric monopoles were sources of electric fields. This was only possible if the product of electric and magnetic charges was quantized. He established the basic relation between the elementary electric charge e and the basic magnetic charge g

$$eg = \frac{n\hbar c}{2}, \text{ where } n = \pm 1, \pm 2, \dots \quad (1)$$

The minimum value of the quantization number is $n = 1$ according to Dirac or $n = 2$ according to Schwinger [5]. However, if the elementary electric charge is considered to be held by the quark then these magnitudes become $n = 3, 6$, respectively. Within this approach, for $n = 1$ and the basic electric charge of the electron, the theoretical minimum magnetic charge is $g_D = 1/2e \simeq 137e/2 = 68.5e$, known as the Dirac magnetic charge. It should be emphasized that magnetic charge, like electric charge, is absolutely conserved, so the lightest magnetically particle is stable, unless annihilated by its antiparticle.

*E-mail: Shoukavy@dragon.bas-net.by

Let us note that the fact of the electric charge quantization is observed with the best accuracy in physics. The experimental expression of the quantization of electric charge is [4]

$$\frac{|Q_e + Q_{\bar{e}}|}{e} < 4 \times 10^{-8}, \quad \frac{|Q_p + Q_{\bar{p}}|}{e} < 1 \times 10^{-8}, \quad \frac{|Q_p + Q_e|}{e} < 1 \times 10^{-21}. \quad (2)$$

where Q_e, Q_p are electron and proton charges, $Q_{\bar{e}}, Q_{\bar{p}}$ are positron and antiproton charges, and e is the electron charge magnitude.

Since the quantization of electric charge in nature is well established but still mysterious, the discovery of just a single monopole would provide a much wanted explanation. Besides explaining the quantization of electric charge, the existence of magnetic charges and of magnetic currents results in the dual symmetrization of Maxwell's equations, and is not forbidden by any known principles of physics.

However, further theoretical development of this idea encountered difficulties. As Dirac's theory involves the singularity line with nonphysical dynamical variables, then Schwinger [6] developed the field theory of electric charges and monopoles excluding the string. It is based on Hamiltonian density expressed nonlocally in terms of field variables. Zwanziger [8, 9] elaborated the local Lagrangian formulation of this theory by using canonical quantization procedure and obtained Feynman rules. However, the use of this formalism in calculations seems problematic, because the coupling constant $\alpha_g \simeq 34.25n^2$ is large and the matrix elements depend explicitly on a space-like vector corresponding to Dirac singularity line [7, 10, 11, 12].

But, after 1974, the monopole problem received the great impetus from 't Hooft and Polyakov works [13, 14]. They independently discovered monopole solutions in the $SO(3)$ Georgi-Glashow model. Then it was demonstrated that any scheme of Grand Unification with the electromagnetic $U(1)$ subgroup embedded into a semi-simple gauge group, which becomes spontaneously broken by Higgs mechanism, possessed monopole solutions. At the same time there were some announcements of magnetic monopole discovery [15] - [17]. In the further, these events have not proved to be true at longer expositions.

The monopoles of the standard Grand Unification must have a mass of the order of the unification scale 10^{17} GeV and therefore cannot be discovered at the current or future accelerators. They could only be produced in the first instants of our Universe and should be searched for in the penetrating cosmic radiation. The most recent search for GUT monopoles in the cosmic radiation was performed by the MACRO detector, using three types of subdetectors (liquid scintillators, limited streamer tubes and nuclear track detectors) [18].

However, in the series of the works (for example [19, 20]) a number of authors showed that the unification scale can be significantly lowered (perhaps even to the TeV scale) through appearance of extra dimensions. Thus, in the models of the Grand Unification the monopoles of masses which can be produced at modern accelerators, without contradictions with slow proton decay [20] can possibly exist.

In conclusion of this section let us underline that the explanation of the quantization of electric charge due to Dirac remains most attractive now. Thus, all these facts stimulate the further experimental search for the magnetic monopoles.

2 The experimental limits on the monopole mass

So, the experimental search for magnetic monopoles at accelerators [21] - [25] and in the penetrating cosmic radiation[18] is continued and will continue.

The magnetic monopoles in the modern multipurpose detectors, such as D0 and CDF (Tevatron), H1 (HERA) or constructing ATLAS and CMS (LHC) can be searched by different ways.

Since the magnetic charge g is large then the monopole makes strong ionization of substance like a heavy nucleus. As we mentioned before the theoretical minimum magnetic charge is $g_D = 1/2e \simeq 137e/2 = 68.5e$. Such a magnetic monopole will make ionization of substance approximately like the thulium ^{69}Tu , but it will go differently in magnetic field. Another method of searching for the monopoles is looking for trapped and bound magnetic monopoles in various accelerator and detectors samples at the time of reparation work[22]. Trapped magnetic monopoles can be draft by magnetic field and registered by the jump of the magnetic field in the SQUID (superconducting quantum interference device) with the passage of the monopoles through it. The main problem of this method to obtain a good screening from external magnetic fields.

Such monopole searches have been realized at Tevatron in $p\bar{p}$ - collisions [23] and HERA [25] in e^+p -collisions. In the experiments at Tevatron it was used the Drell-Yan monopole-antimonopole pair production and the method suggested Ginsburg and collaborators . The main idea of this method is based on observation that the interaction strength between monopole and photon is very strong and could give rise to photon-photon rescattering via virtual monopole loop. First there was in 1998 D0 group search for the virtual production of monopoles [21], based on the theory of Ginzburg and collaborators [26] - [29]. In 2000 and 2004 results from an experiment (Fermilab E882) searching for real magnetically charged particles bound to elements from the CDF and D0 detectors were reported [22, 23]. The strongest direct experimental limits, from the CDF collaboration, have been obtained in 2005. Less strong, but complementary, limits from the H1 collaboration at HERA were published in the same year [25], using the other mechanism for magnetic monopole production. The absence of the monopole signal on the basis of assumptions about the monopole pair production mechanisms is treated as limits on the mass and production cross section of Dirac monopole. The most recent limits on the monopole mass are given Fig.1

HERA	Tevatron	Tevatron, CDF Run II
Direct search	Direct search, Experiment E-882	Drell-Yan pair production mechanism
$\sqrt{s} = 300 \text{ GeV}$	$\sqrt{s} = 1,8 \text{ GeV}$	$\sqrt{s} = 1,96 \text{ GeV}$
$L = 62 \pm 1 \text{ pb}^{-1}$	$L = 172 \pm 8 \text{ pb}^{-1}$	$L = 35,7 \text{ pb}^{-1}$
$ n =1,2,3,6$	$ n =1, \text{ M} > 285 \text{ GeV}$	$ n =1$
$\text{M} > 140 \text{ GeV}$	$ n =2, \text{ M} > 355 \text{ GeV}$	$\text{M} > 360 \text{ GeV}$
	$ n =3, \text{ M} > 325 \text{ GeV}$	
	$ n =6, \text{ M} > 420 \text{ GeV}$	

Figure 1: The modern experimental limits on the monopole mass

It is seen from Fig.1 that in these different accelerators and detectors close limits on the monopole mass of order several hundred GeV are obtained. In this table we don't include the limits on the Dirac monopole mass which was obtained at the Tevatron (D0 collaboration) from the analysis of the process for $\gamma\gamma$ production via virtual monopole loop [21], because they are strongly criticized and questioned now [30, 31, 32]. Also let us mention about interesting indirect limits on monopole mass from experimental data on electron electric dipole moment [34].

It should be emphasized that if GUT monopole have the small mass of order TeV then it does not differ from Dirac monopole by its detection characteristics in the mentioned experiments. Therefore, from the present experiments at accelerations the limits on the mass of magnetic monopoles of any nature are followed.

3 Two-photon production of monopole–antimonopole pair

At present only Drell-Yan mechanism for magnetic monopole pair production was usually used for search for magnetic monopoles expect in the HERA experiment. We would like to propose other alternative mechanism for magnetic monopole production. It is two-photon production mechanism.

Schematic diagrams for these two mechanisms are given on Fig.2 and Fig.3,6. They give very rough indication of cross section values because strong interaction between monopoles and photon.

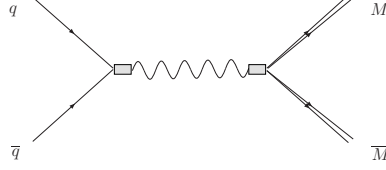


Figure 2: Schematic diagram for Drell-Yan monopole antimonopole pair production mechanism (monopole spin $s = 1/2$).

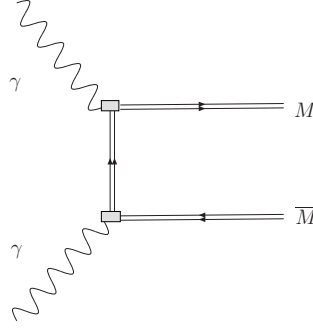


Figure 3: Schematic diagram for monopole antimonopole pair $\gamma\gamma$ production mechanism (monopole spin $s = 1/2$).

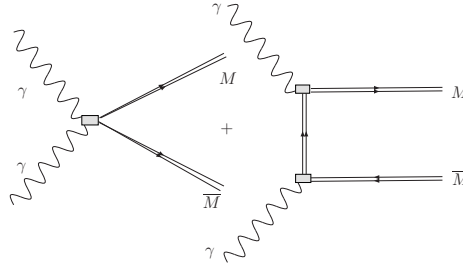


Figure 4: Schematic diagram for monopole antimonopole pair $\gamma\gamma$ production mechanism (monopole spin $s = 0, 1$).

For the general case of two-photon collisions the cross section for the reaction $A_1 A_2 \longrightarrow A'_1 A'_2 X_f$ in a very good approximation can be factorized into the photon spectrum $f_{\gamma/p}(z)$ and the cross section of the photon-photon interaction process $\gamma\gamma \longrightarrow X_f$, where A_1, A_2 are the

initial particles, A'_1, A'_2 are their final states after the photon emission, and X_f is the final state produced in photon-photon collision [35].

Thus, in our case the total two-photon production cross section for monopole-antimonopole pair is written as

$$\sigma_{pp \rightarrow M\overline{M}}(s) = \int_{4M^2/s}^1 dz_1 \int_{4M^2/s}^1 dz_2 f_{\gamma/p}(z_1) f_{\gamma/p}(z_2) \sigma_{\gamma\gamma}(\hat{s} = z_1 z_2 s), \quad (3)$$

$\sqrt{s} = 14$ TeV, M is a monopole mass.

In this paper we consider the case of elastic production. In the case of nonelastic production the cross section is even larger. But, we don't consider it here. For elastic photon spectrum $f_{\gamma/p}^{el}(z)$ for pp we use an approximate analytic expression given in [36] which is known to reproduce exact results to about 10%. The form we use is given by

$$f_{\gamma/p}(z) = \frac{\alpha}{2\pi z} (1 + (1-z)^2) \left[\ln A - \frac{11}{6} + \frac{3}{A} - \frac{3}{2A^2} + \frac{1}{3A^2} \right], \quad (4)$$

where

$$A = 1 + \frac{0.71(\text{GeV})^2}{Q_{min}^2}, \quad (5)$$

and

$$Q_{min}^2 = -2m_p^2 + \frac{1}{2s} \left[(s + m_p^2)(s - zs + m_p^2) - (s - m_p^2) \sqrt{(s - zs - m_p^2)^2 - 4m_p^2 zs} \right]. \quad (6)$$

At high energy Q_{min}^2 is given to a very good approximation by $m_p^2 z^2 / (1-z)$.

Because the monopole-photon coupling is large and the theory non-perturbative, there is no universally accepted field-theoretic calculation of magnetic monopole production. But, for electron-monopole scattering this cross section differs from the Rutherford one for electron-electron scattering by the replacement (see[32, 37])

$$\frac{e}{v} \rightarrow \frac{g}{c}. \quad (7)$$

Perhaps, monopole interactions with matter, such as scattering or annihilation, require only a replacement of electric charge with the monopole's effective charge $g\beta$ (where β is the velocity of the monopole).

In our calculations we consider an $n = 1$ monopole with mass up to 2 TeV, spin $s = 0, 1/2, 1$ and we make the naive replacement $e \rightarrow g\beta$ in the total cross section for pair production of magnetic monopoles via $\gamma\gamma$ fusion.

Thus, the total cross section $\sigma_{\gamma\gamma \rightarrow M\overline{M}}$ for the subprocess $\gamma\gamma \rightarrow M\overline{M}$ for monopole spins $s = 0, 1/2, 1$ may be written respectively as

$$\sigma_{\gamma\gamma}^{s=0}(\hat{s}) = \frac{4\pi\alpha_g^2}{\hat{s}} \beta \left[2 - \beta^2 - \frac{1 - \beta^4}{2\beta} \ln \left(\frac{1 + \beta}{1 - \beta} \right) \right], \quad (8)$$

$$\sigma_{\gamma\gamma}^{s=1/2}(\hat{s}) = \frac{4\pi\alpha_g^2}{\hat{s}} \beta \left[\frac{3 - \beta^4}{2\beta} \ln \left(\frac{1 + \beta}{1 - \beta} \right) - 2 + \beta^2 \right], \quad (9)$$

and

$$\sigma_{\gamma\gamma}^{s=1}(\hat{s}) = \frac{\pi\alpha_g^2}{\hat{s}} \beta \left[-3 \frac{1 - \beta^4}{\beta} \ln \left(\frac{1 + \beta}{1 - \beta} \right) + 2 \frac{22 - 9\beta^2 + 3\beta^4}{1 - \beta^2} \right], \quad (10)$$

where

$$\alpha_g = g^2 \beta^2, \quad \beta = \sqrt{1 - 4M^2/\hat{s}}. \quad (11)$$

As is well known (for example [32]), if the cross section were dominated by a single partial wave of angular momentum J , the cross section would be bounded by

$$\sigma_J \leq \frac{4\pi}{s}(2J+1) \quad (12)$$

Comparing this with the our cross sections given above (8)-(10), we obtain at large values β that these cross sections violates unitarity relation. At the same time, at small values β the cross section (8)-(10) satisfy the unitarity relation. Because at small values β we effectively reduce a value of the coupling constant of a photon to a monopole α_g (see (11)). It means that we need to use some kind of form-factor $F_{TW}(\hat{s})$ depending on energy for $\gamma\gamma$ -monopole interaction.

We need to impose following restrictions on form-factor $F(\hat{s})$

$$F_{TW}^4(\hat{s})\alpha_g^2 \leq 1 \quad (13)$$

that our cross sections satisfy the unitarity relation.

Unfortunately this form-factor cannot be obtained theoretically now. In this paper for us it is enough to use as form-factor the number 0.17 at each g for an estimation two-photon production cross section for monopole-antimonopole pair. In the result, we have no any contradiction with unitarity for $\gamma\gamma$ -processes for all values β , but underestimate cross sections.

It should be emphasized that one will have a similar situation for Drell-Yan monopole - antimonopole production mechanism with following condition

$$F_{DY}^2(\hat{s})\alpha_g \leq 1. \quad (14)$$

It is seen from Fig.7 that total two-photon production cross section for monopole-antimonopole pair $\sigma_{pp \rightarrow M\bar{M}}$ quickly decreases with the increase of the monopole mass. Also from Fig.7 one observes that cross sections for pair production of magnetic monopoles via $\gamma\gamma$ fusion grow with spin value.

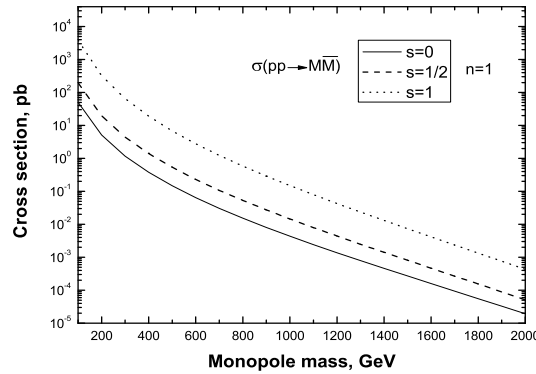


Figure 5: The comparison total $\gamma\gamma$ production cross section for monopole antimonopole pair as function monopole mass for monopoles of spin $s = 0, 1/2, 1$.

It is not excluded that for the different spin monopoles the reasons of discrete P - and C -symmetries can strongly inhibit the Drell-Yan pair production mechanism [38] or our mechanism. However, such reasons can leave without the significant changes one of two possible mechanisms.

So, two-photon mechanism for production of monopole-antimonopole pair must be taken into consideration using of the two-photon production of monopole-antimonopole pair simultaneously with Drell-Yan production mechanism will reflect more precisely the search for magnetic monopoles in the absence thereof the consistent quantum-field theory.

4 Monopole mass limit from two-photon pair production mechanism

As in CDF results [24] only Drell-Yan mechanism for magnetic monopole pair production was used, we would like to estimate the influence of two-photon mechanism on mass limits.

Further, we will use the results and notations of the work [24]. In our calculations we consider an $n = 1$ monopole with mass up to 1 TeV, spin $s = 1/2$ and we make the naive replacement $e \rightarrow g\beta$ in the total cross section for pair production of magnetic monopoles via $\gamma\gamma$ fusion. In our case the total cross-section $\sigma_{\gamma\gamma \rightarrow M\bar{M}}$ has a form (3), where $\sigma_{\gamma\gamma}(\hat{s} = z_1 z_2 s)$ given by (9).

Let us underline that in this section we apply the scheme of calculation which was used in [24]. In this approach we don't use the unitarity condition for estimation of the total cross section.

The comparison of the $\gamma\gamma$ production cross section for monopole-antimonopole pair and Drell-Yan cross section at Tevatron energies is given in Fig.2. We find that $\gamma\gamma$ production cross section for monopole-antimonopole pair even for elastic case is larger then Drell-Yann cross section (Fig.6) for monopole mass up to 600 GeV, if one believes in perturbative estimations.

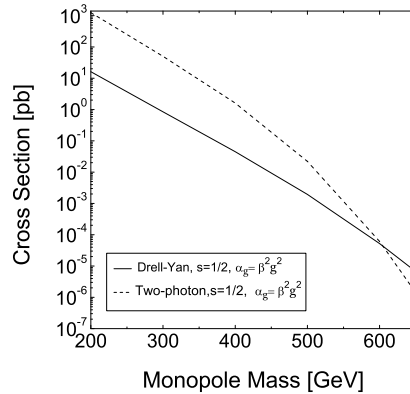


Figure 6: The comparison of the $\gamma\gamma$ production cross section for monopole-antimonopole pair and Drell-Yan cross section in $p\bar{p}$ -collisions at $\sqrt{s} = 1.96$ TeV.

Recently, CDF collaboration [24] found a mass limit $M > 360$ GeV for the Drell-Yan pair production mechanism. For two-photon production mechanism we have a mass limit of 455 GeV in idealized case of similar registration efficiencies for two mechanisms, as it follows from Fig.7. This increase of mass limit is explained by the fact that the predicted two-photon total cross section for monopole-antimonopole pair is larger then Drell-Yann cross section for monopole mass up to 600 GeV.

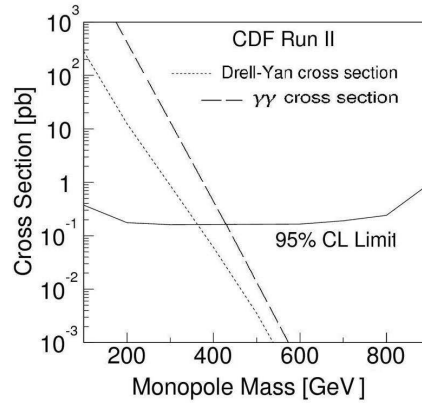


Figure 7: The 95% CL cross-section upper limit versus magnetic monopole mass (see [24]). The dot line is a curve for Drell-Yan monopole pair production intersects at the mass limit $M > 360$ GeV [24]. The dash line is our estimation for $\gamma\gamma$ monopole pair production. It intersects at the mass limit $m > 455$ GeV.

5 Conclusion

Magnetic monopoles are inevitable features of current unification theories. In some Grand Unified scenarios values of monopole mass which can be produced at LHC energies are allowed. While, the Dirac magnetic monopole does not have strict theoretical limits on the mass at all. So, all this stimulates the experimental search for the magnetic monopoles at every new accelerator.

It is necessary to take into consideration also the two-photon production of monopole-antimonopole pair for the search for magnetic monopole at LHC. We analyze the total two-photon production cross section for monopole antimonopole pair for spin monopole $s = 0, 1/2, 1$ at LHC energies. It is shown in duality approach, that the detection probability for monopole in the case of two-photon production mechanism is comparable with Drell-Yan one and quickly decreases with the increase of the monopole mass.

This new mechanism was investigated by us in $p\bar{p}$ -collisions at Tevatron energies (CDF). We received the following results:

(a) $\gamma\gamma$ production cross section for monopole-antimonopole pair is larger then Drell-Yann cross section for monopole mass up to 600 GeV, in idealized case of similar registration efficiencies for two mechanisms. The investigation of real registration efficiency for two photon mechanism will be important for us only in LHC monopole search where Drell-Yan mechanism will have own drawback.

(b) CDF collaboration for the Drell-Yan pair production mechanism have been found a mass limit $M > 360$ GeV. In the case of the two-photon production mechanism we had a mass limit $M > 455$ GeV. So, real comparison corresponding cross section with experimental ones doesn't give exact limits for monopole masses. It means that the real experimental results for monopole mass is no more than 0,5 TeV that corresponds to the purely kinematical limits for monopole appearance.

References

- [1] Pierre de Maricourt, On the Magnet, letter to Siger de Foucaucourt, (1269). In The Letter of Petrus Peregrinus on the Magnet, New York, 1904. McGraw-Hill. Translated by Brother Arnold.
- [2] P. Curie, On the possible existence of magnetic conductivity and free magnetism. *Sèances Soc. Phys. (Paris)*, **76** (1894).
- [3] P.L.M. Dirac, *Proc. Roy. Soc.* **A133**, 60 (1934).
- [4] S. Eidelman et al. (PDG Collab.), *Phys. Lett. B* **592**, 33, 67 (2004).
- [5] L.J. Schwinger, *Phys. Rev.* **D12**, 3105 (1975).
- [6] L.J. Schwinger, *Phys. Rev.* **144**, 1087; **151**, 1048; **151**, 1055 (1966).
- [7] D. Zwanzinger, *Phys. Rev. D* **33**, 880 (1971).
- [8] R. Brandt, F.Nerri and D. Zwanzinger, *Phys. Rev. Lett.* **40**, 147 (1978).
- [9] R. Brandt, F.Nerri and D. Zwanzinger, *Phys. Rev. D* **19**, 1153 (1979).
- [10] M. Blagojevic and P. Senjanovic, *Phys. Rep.* **157**, 233 (1988).
- [11] V.I. Strazhev and L.M. Tomilchik, *Electrodynamics with Magnetic Charge* (Nauka and Tekhnika, Minsk, 1975) [in Russian].
- [12] Ya. Shnir, *Magnetic Monopoles* (Springer-Verlag Berlin Heidelberg, 2005).
- [13] G.'t Hooft, *Nucl. Phys. B* **79**, 276 (1974).
- [14] A.M. Polyakov, *JETP Letters* **20**, 430 (1974).
- [15] P. B. Price, E. K. Shirk, W. Z. Osborne, and L. S. Pinsky, *Phys. Rev. Lett.* **35**, 487 (1975).
- [16] B. Cabrera, *Phys. Rev. Lett.* **48**, 1378 (1982).
- [17] A.D. Caplin et al., *Nature* **321**, 402 (1986).
- [18] M. Ambrosio et al. (MACRO Collab.), *Eur. Phys. J. C* **25**, 511 (2002).
- [19] K.R. Gienes, E. Dudas and T. Gherghetta *Phys. Rev. B* **436**, 55 (1998).
- [20] V.A. Rubakov, *UFN* **171**, 913 (2001)[in Russian].
- [21] B. Abbott et al. (D0 Collab.), *Phys. Rev. Lett.* **81**, 524 (1998).
- [22] G.R. Kalbfleisch, K.A. Milton, M.G. Strauss, L. Gamberg, E.H. Smith, and W. Luo, *Phys. Rev. Lett.* **85**, 5292 (2000).
- [23] G.R. Kalbfleisch, W. Luo, K.A. Milton, E.H. Smith, M.G. Strauss, *Phys.Rev. D* **69**, 052002 (2004).
- [24] CDF Collab. (A. Abulencia et al.), *Phys. Rev. Lett* **96**, 201801 (2006).
- [25] A. Aktas et al. (H1 Collab.), *Eur. Phys. J. C* **41**, 133 (2005).

- [26] I.F. Ginzburg and S.L. Panfil, Yad. Fiz. **36**, 1461 (1982).
- [27] A. De Ginzburg and A. Shiller, Phys. Rev. D **57**, R6599 (1998).
- [28] I.F. Ginzburg and A. Shiller, Phys. Rev. D **60**, 075016 (1999).
- [29] A. De Rujula, Nucl. Phys. B **435**, 609 (1995).
- [30] L. Gamberg, G.Kalbfleisch, K. Milton, Found. Phys. **30**, 543 (2000).
- [31] K. Milton, G. Kalbfleisch, W. Luo, L. Gamberg, Int. J. Mod. Phys. A **17**, 732 (2002).
- [32] K. A. Milton, Rep. Prog. Phys. **69**, 1637 (2006).
- [33] S.G. Kovalevich, P. Osland, Ya.M. Shnir and E.A. Tolkachev, Phys. Rev. D **55**, 5807 (1997).
- [34] S.G. Kovalevich, P. Osland, Ya.M. Shnir and E.A. Tolkachev, Phys. Rev. D **55**, 5807 (1997).
- [35] M. Drees, R.M. Godbole, M. Nowakovsky and S.D. Rindani, Phys.Rev. D **50**, 2335 (1994).
- [36] M. Drees, D. Zeppenfeld, Phys. Rev. D **39**, 2536 (1989).
- [37] J. Schwinger, K. A. Milton, W.-y. Tsai, L. L. DeRaad, Jr., and D. C. Clark, Ann. Phys. (N.Y.) **101**, 451 (1976).
- [38] A.Yu.Ignatiev, G.C.Joshi, Chaos, Solitons and Fractals **11**, 1411 (2000).